International Journal of Latest Trends in Engineering and Technology
Vol.(10)Issue(2), pp.067-070
DOI: http://dx.doi.org/10.21172/1.102.10
e-ISSN:2278-621X

A SHORT NOTE ON THE ORIGIN OF THE TURBULENT BOUNDARY LAYER

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Abstract- This paper reports on an investigation of the influence of leading edge location (of a flat plate) on skin friction in a zero pressure gradient turbulent boundary layer. An attempt has been made to correct the turbulent boundary layer data in accordance with virtual leading edge of the turbulent boundary layer so that the data of different sources can be compared. Experimental data of two different sources are used for analysis. Encouraging results are obtained from the present approach.
Keywords – Turbulent boundary layer, Reynolds number, skin friction, leading edge.

1. INTRODUCTION

In experimental work of turbulent boundary layers, the flat plate is placed in a plane flow with an oncoming fluid velocity U_o. A laminar boundary layer starts developing from the leading edge. At this leading edge, the flow exhibits a singularity. Further downstream from the leading edge, the laminar flow becomes unstable and a turbulent boundary layer starts developing. There exists a region of transition between laminar and turbulent flow. Unfortunately no transition Reynolds number ($\mathrm{Re}_{transition}$) can be defined for the location of the transition zone. That is the practical reason for introduction of a tripping device. In addition of that one saves the space needed for a long entry length to develop turbulent flow. Usually from the tripping device, downstream position x is measured. The downstream location of a profile is usually expressed through the local Reynolds number Re_x defined as

$$\mathrm{Re}_x = \frac{U_o x}{\nu} \tag{1}$$

where ν is kinematic viscosity of fluid.

2. THE DATA: WIEGHARDT AND TILLMANN (PERSEN [1]) AND OSTERLUND [2]

The data from two different sources have been considered for analysis. These experimental data have been duly exposed to the profession through published papers. The best results on a flat plate are thought to be the ones obtained by Wieghardt and Tillmann (Persen [1]) and which are written out in detail in the Proceedings of the Stanford Conference, Coles and Hirst [3]. These data are scrutinized by Coles, and they are obtained by means of a total head probe rake. These data incorporate measurements of profiles located at small values of x. First profile is located at $x = 0.187m$. The experiments incorporates only one oncoming velocity, $U_o \approx 33m/s$.

Osterlund [2] measurements incorporate profiles measured at different oncoming speeds U_o. The speed is presumably set at 10 different levels, $\approx 10.4 - 53.4m/s$ and they are obtained with hot-wire/hot-film anemometry. The measurement starts from downstream distance $x = 1.5m$.

3. RESULTS AND DISCUSSION

Local skin friction, c_f for zero pressure gradient flow can be related to non-dimensional oncoming velocity $\xi = U_o / v_*$.

$$c_f = \frac{1}{\xi^2 / 2}$$

where v_* is shear velocity.

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Though U_o is kept constant for an experimental run but non-dimensional oncoming velocity ξ is a function of downstream distance x. It happens due to the fact that v_* is a decaying function of x. Figure 1 shows variation of $\xi^2/2$ with downstream distance, x.

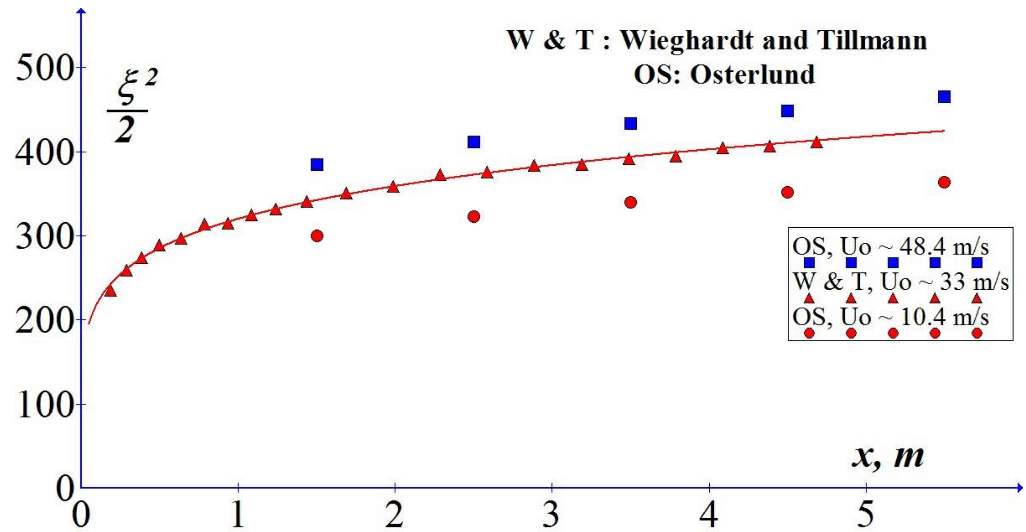

Figure 1. The values $1/c_f = \xi^2/2$ plotted as function of downstream distance.

The data illustrate that $1/c_f = \xi^2/2 \to 0$ as $x \to 0$ as it should do at a leading edge. (well understood from the Wieghardt and Tillmann data). From these data, it is very difficult to determine the virtual leading edge of the flat plate, $x = x_o$ where $\xi^2/2 = 0$ by extrapolation because Osterlund [2] measured the profiles located far away from the leading edge. In spite of that an attempt has been made by Persen [4] to determine virtual leading edge of the flat plate by extrapolation using Osterlund data [2]. In the present paper a new approach is proposed to solve this origin problem.

Researchers and engineers preferred dimensionless plot of different turbulent boundary layer variables. Figure 2 below shows such a plot between local skin friction c_f and local Reynolds Numbers Re_x.

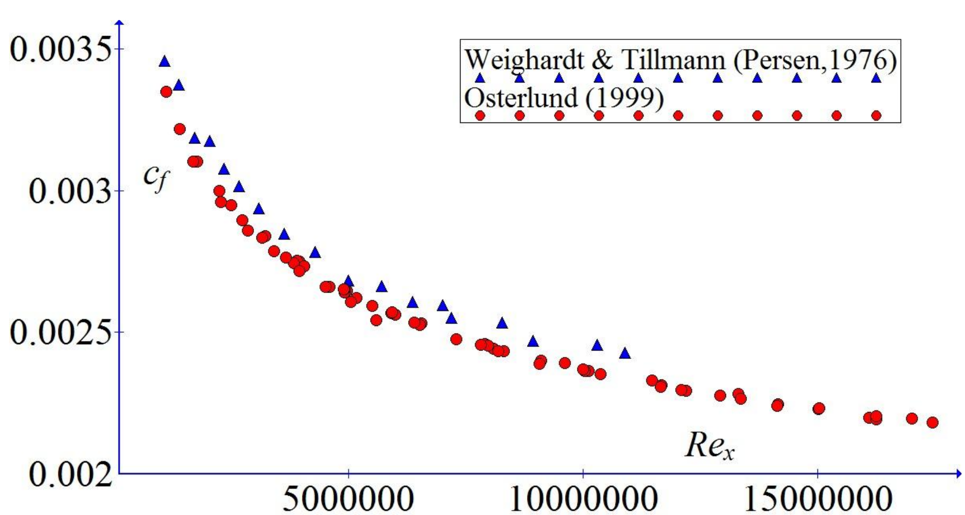

Figure 2 Plot of c_f vs Re_x

It can be seen that two data from different sources form two separate curves. One may simply attribute the cause of difference mainly to their ways of fixing the downstream distance, x.
In view of above it is clear that the downstream position must be uniquely defined so that the results from different sources can be compared. That is of course not the case with the arbitrarily chosen position of the tripping device from which the downstream distance is measured in the laboratory.
The Von Karman momentum integral equation obtained from the Reynolds-averaged stream wise momentum equation for zero pressure gradient (ZPG) turbulent boundary layers is given by

$$\frac{c_f}{2} = \frac{d\theta}{dx'} = \frac{d\,\mathrm{Re}_\theta}{d\,\mathrm{Re}_{x'}} = \frac{{v_*}^2}{{U_0}^2} = \frac{1}{\xi^2} \tag{2}$$

where θ is the momentum thickness, x' is the stream wise distance from a corrected origin which accounts for the length of initial laminar boundary layer and /or the location of virtual origin, $\mathrm{Re}_\theta \left(= U_0\theta/\nu\right)$ is the momentum thickness Reynolds number, $\mathrm{Re}_{x'} \left(= U_o x'/\nu\right)$ is the local Reynolds number based on x' and other variables are defined earlier.
For ZPG turbulent boundary layers ξ can be expressed as

$$\xi = A\ln\left(\mathrm{Re}_\theta\right) + B \tag{3}$$

where A and B are constants.
Substituting equation (3) in equation (2) one easily obtains

$$\frac{d\,\mathrm{Re}_\theta}{d\,\mathrm{Re}_{x'}} = \left[A\ln\left(\mathrm{Re}_\theta\right) + B\right]^{-2} \tag{4}$$

The inverse of equation (4) can be integrated to yield

$$\mathrm{Re}_{x'} = \mathrm{Re}_\theta\left[(\xi - A)^2 + A^2\right] \tag{5}$$

From Osterlund’s data constants A and B are found as

$$A = 2.5981 \text{ and } B = 4.0802 \tag{6}$$

Substituting equation (6) into equation (5)

$$\mathrm{Re}_{x'} = \mathrm{Re}_\theta\left[(\xi - 2.5981)^2 + 6.7503\right] \tag{7}$$

From Osterlund’s data relation between Re_θ and Re_x (based laboratory measured distance) is obtained as

$$\mathrm{Re}_\theta = 0.0250\,\mathrm{Re}_x^{\ 0.8258} \tag{8}$$

Substituting equation (8) into equation (7) one obtains

$$\mathrm{Re}_{x'} = 0.0250\,\mathrm{Re}_x^{\ 0.8258}\left[(\xi - 2.5981)^2 + 6.7503\right] \tag{9}$$

From Wieghardt / Tillmann data following relations are obtained:

$$\mathrm{Re}_{x'} = \mathrm{Re}_\theta\left[(\xi - 2.4831)^2 + 6.1656\right] \tag{10}$$

$$\mathrm{Re}_\theta = 0.0161\,\mathrm{Re}_x^{\ 0.8502} \tag{11}$$

$$\mathrm{Re}_{x'} = 0.0161\,\mathrm{Re}_x^{\ 0.8502}\left[(\xi - 2.4831)^2 + 6.1656\right] \tag{12}$$

For known values of Re_x and ξ, $\mathrm{Re}_{x'}$ can be calculated from equation (9) for Osterlund’s case and $\mathrm{Re}_{x'}$ can be calculated from equation (12) for Wieghardt/Tillmann case.
Coefficients of skin friction obtained from two experiments are plotted as a function of $\mathrm{Re}_{x'}$ in Figure 3. The data are plotted in similar scale as in Figure 2. It can be revealed that the present analysis remove the difference between the data in a great extent.

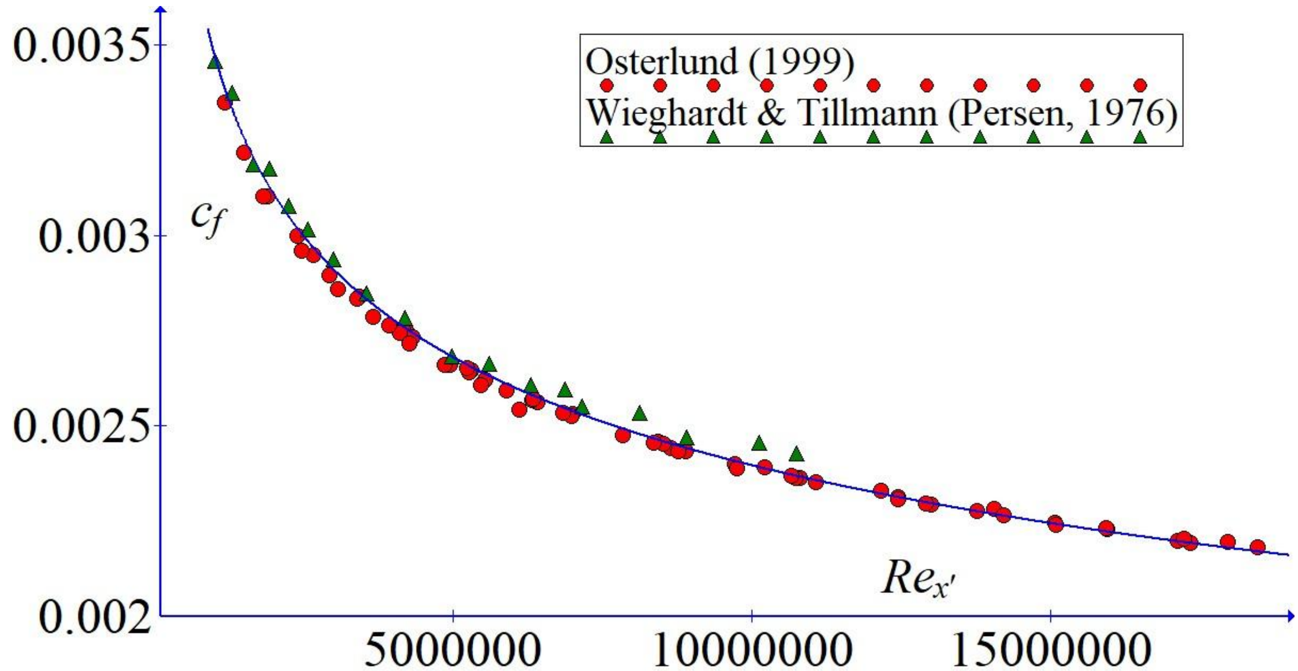

Figure 3 Corrected plot: c_f vs $\mathrm{Re}_{x'}$

4.CONCLUSION

Development of any theoretical approach depends on a proper definition of the initial conditions of the problem. Due improper selection of actual leading edge, the data from different sources cannot be compared. The present approach reveals that with a correction of local Reynolds Numbers, the Wieghardt and Tillmann data (Persen. [1]) and the Osterlund data [2] are amazingly close. It is also observed that the corrected Reynolds Numbers for Wieghardt and Tillmann data are lesser than that reported earlier while corrected Reynolds Numbers for Osterlund data are higher than that reported earlier. It implies that in Wieghardt and Tillmann experiment, virtual leading edge was located at the right hand side of the tripping point. On the contrary, in Osterlund experiment, virtual leading edge was located at the left hand side from the tripping point.

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